D_1 et D_2 sont coplanaires ssi $\det(u, v, \overrightarrow{AB}) = 0$.
$d(M, D_1) = \dfrac{\|\overrightarrow{AM} \wedge u\|}{\|u\|}$.
Le centre (éventuel) d'une quadrique d'équation implicite $f(x, y, z) = 0$ est la (ou une) solution de $gradf = 0$.
L'équation réduite d'un paraboloïde hyperbolique est $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = z$ (la matrice associée est de rang 2 avec 2 valeurs propres non nulles de signes opposés, et la partie affine après rotation est de rang 1 ; $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ donnerait un cylindre hyperbolique, $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 0$ la réunion de 2 plans)

O20-113

```
> restart;
> with(LinearAlgebra):
> u:=<2,2,1>;v:=<1,2,2>;a:=<2,3,4>;b:=<-1,1,1>;ab:=b-a;
```

$$u := \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

$$v := \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$$a := \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$$b := \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$ab := \begin{bmatrix} -3 \\ -2 \\ -3 \end{bmatrix}$$

```
> m:=Matrix([u,v,ab]);Determinant(m);
```

$$m := \begin{bmatrix} 2 & 1 & -3 \\ 2 & 2 & -2 \\ 1 & 2 & -3 \end{bmatrix}$$

$$-6$$

```
> p1:=convert(a-4*u,list);p2:=convert(b-4*v,list);q1:=convert(a+4*
  u,list);q2:=convert(b+4*v,list);
```

$$p1 := [-6, -5, 0]$$

$$p2 := [-5, -7, -7]$$

$$q1 := [10, 11, 8]$$

$$q2 := [3, 9, 9]$$

```
> with(plots):
Warning, the name changecoords has been redefined

> droites:=polygonplot3d([[p1,q1],[p2,q2]]):
> d1:=Norm(CrossProduct(<x-2,y-3,z-4>,u),2)/Norm(u,2);
```

$$d1 := \frac{1}{3}\sqrt{|-y-5+2z|^2+|-2z+6+x|^2+|2x+2-2y|^2}$$

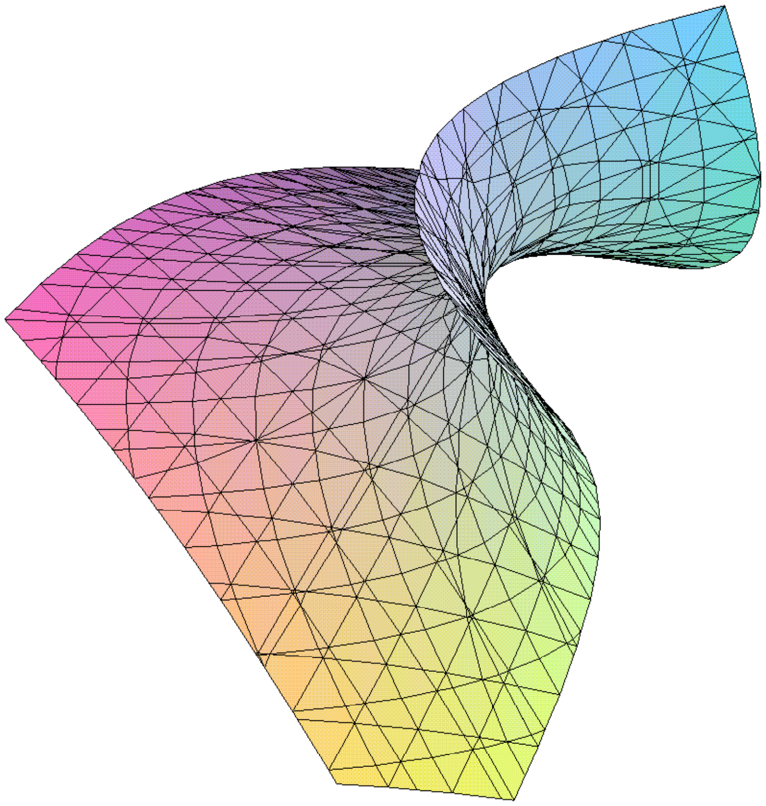
```
> d2:=Norm(CrossProduct(<x+1,y-1,z-1>,v),2)/Norm(v,2);
```

$$d2 := \frac{1}{3}\sqrt{|-2y+2z|^2+|-z+3+2x|^2+|2x+3-y|^2}$$

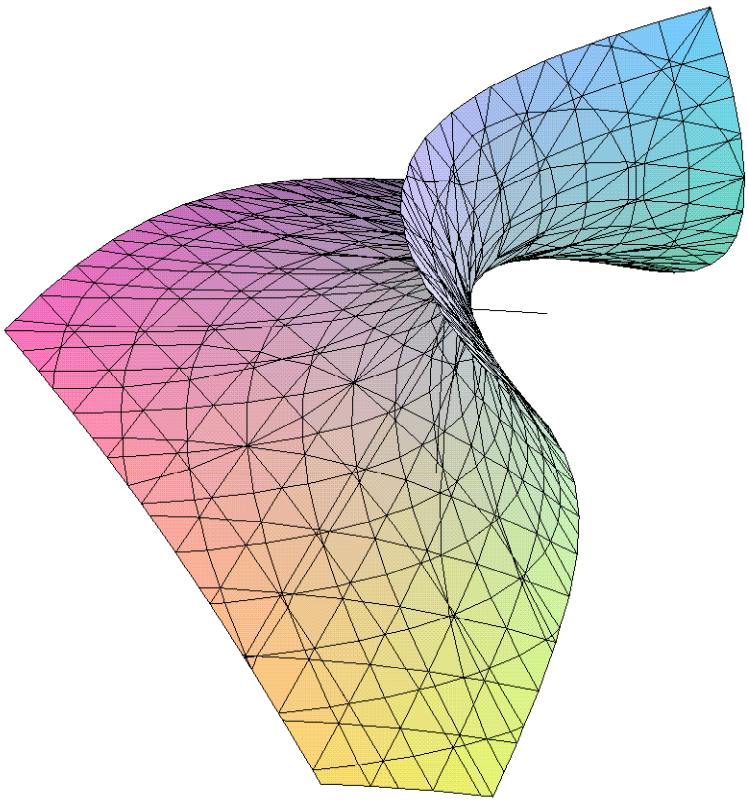
```
> assume(x::real,y::real,z::real):
> s:=simplify(d1^2-d2^2);
```

$$s := -\frac{4}{9}x\sim - \frac{38}{9}z\sim + \frac{8}{9}y\sim + \frac{4}{9}y\sim z\sim + \frac{1}{3}z\sim^2 - \frac{1}{3}x\sim^2 - \frac{4}{9}x\sim y\sim + \frac{47}{9}$$

```
> surf:=implicitplot3d(
  s,x=-10..10,y=-10..10,z=-10..10,grid=[13,13,13]):surf;
```



```
> display([droites,surf]);
```



```
> q:=Matrix([[-1/3,-2/9,0],[-2/9,0,2/9],[0,2/9,1/3]]);
```

$$q := \begin{bmatrix} \frac{-1}{3} & \frac{-2}{9} & 0 \\ \frac{-2}{9} & 0 & \frac{2}{9} \\ 0 & \frac{2}{9} & \frac{1}{3} \end{bmatrix}$$

```
> red:=Eigenvectors(q);p1:=red[2];diag:=red[1];
```

$$red := \begin{bmatrix} 0 \\ \frac{\sqrt{17}}{9} \\ -\frac{\sqrt{17}}{9} \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 \\ \frac{-3}{2} & -\frac{\sqrt{17}}{2} - \frac{3}{2} & \frac{\sqrt{17}}{2} - \frac{3}{2} \\ 1 & -\frac{13}{4} - \frac{3\sqrt{17}}{4} & -\frac{13}{4} + \frac{3\sqrt{17}}{4} \end{bmatrix}$$

$$p1 := \begin{bmatrix} 1 & 1 & 1 \\ \frac{-3}{2} & -\frac{\sqrt{17}}{2}-\frac{3}{2} & \frac{\sqrt{17}}{2}-\frac{3}{2} \\ 1 & -\frac{13}{4}-\frac{3\sqrt{17}}{4} & -\frac{13}{4}+\frac{3\sqrt{17}}{4} \end{bmatrix}$$

$$diag := \begin{bmatrix} 0 \\ \frac{\sqrt{17}}{9} \\ -\frac{\sqrt{17}}{9} \end{bmatrix}$$

```
> baseortho:=simplify([seq(Column(p1,i)/Norm(Column(p1,i),2),i=1..
  3)]);
```

$$baseortho := \left[\begin{bmatrix} \frac{2\sqrt{17}}{17} \\ -\frac{3\sqrt{17}}{17} \\ \frac{2\sqrt{17}}{17} \end{bmatrix}, \begin{bmatrix} \frac{4}{17+3\sqrt{17}} \\ -\frac{2(\sqrt{17}+3)}{17+3\sqrt{17}} \\ -\frac{13+3\sqrt{17}}{17+3\sqrt{17}} \end{bmatrix}, \begin{bmatrix} -\frac{4}{-17+3\sqrt{17}} \\ -\frac{2(\sqrt{17}-3)}{-17+3\sqrt{17}} \\ -\frac{-13+3\sqrt{17}}{-17+3\sqrt{17}} \end{bmatrix}\right]$$

```
> p:=Matrix(baseortho);
```

$$p := \begin{bmatrix} \frac{2\sqrt{17}}{17} & \frac{4}{17+3\sqrt{17}} & -\frac{4}{-17+3\sqrt{17}} \\ -\frac{3\sqrt{17}}{17} & -\frac{2(\sqrt{17}+3)}{17+3\sqrt{17}} & -\frac{2(\sqrt{17}-3)}{-17+3\sqrt{17}} \\ \frac{2\sqrt{17}}{17} & -\frac{13+3\sqrt{17}}{17+3\sqrt{17}} & -\frac{-13+3\sqrt{17}}{-17+3\sqrt{17}} \end{bmatrix}$$

```
> X:=p.<x1,y1,z1>;
```

$$X := \begin{bmatrix} \frac{2\sqrt{17}\,x1}{17}+\frac{4\,y1}{17+3\sqrt{17}}-\frac{4\,z1}{-17+3\sqrt{17}} \\ -\frac{3\sqrt{17}\,x1}{17}-\frac{2(\sqrt{17}+3)\,y1}{17+3\sqrt{17}}-\frac{2(\sqrt{17}-3)\,z1}{-17+3\sqrt{17}} \\ \frac{2\sqrt{17}\,x1}{17}-\frac{(13+3\sqrt{17})\,y1}{17+3\sqrt{17}}-\frac{(-13+3\sqrt{17})\,z1}{-17+3\sqrt{17}} \end{bmatrix}$$

```
> s1:=sort(simplify(subs({x=X[1],y=X[2],z=X[3]},s)));
```

$$s1 := -\frac{1}{9}\sqrt{17}\,z1^2+\frac{1}{9}\sqrt{17}\,y1^2+\frac{17}{9}z1-\frac{47}{153}\sqrt{17}\,z1-\frac{12}{17}\sqrt{17}\,x1+\frac{47}{153}\sqrt{17}\,y1+\frac{17}{9}y1+\frac{47}{9}$$

```
> xc:=solve(diff(s1,z1),z1);yc:=solve(diff(s1,y1),y1);
```

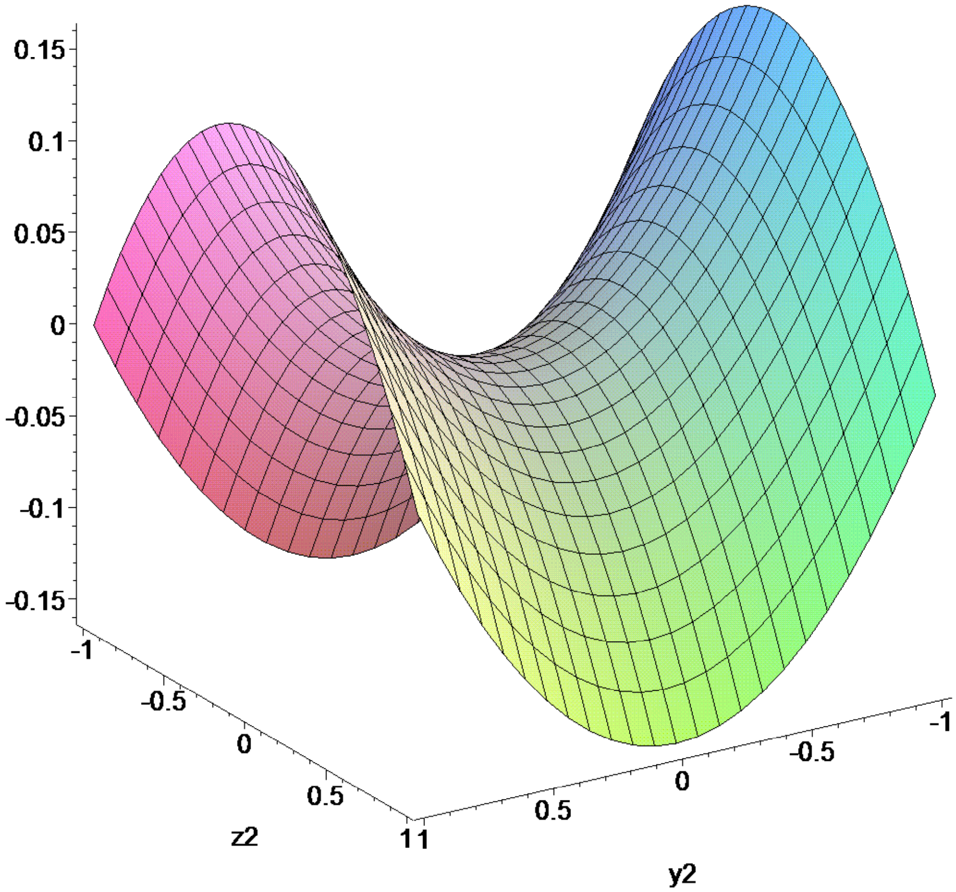
$$xc := -\frac{(-289+47\sqrt{17})\sqrt{17}}{578}$$

$$yc := -\frac{(47\sqrt{17}+289)\sqrt{17}}{578}$$

```
> s2:=sort(simplify((9/sqrt(17))*subs({z1=z2+xc,y1=y2+yc},s1)));
```

$$s2 := -z2^2 + y2^2 - \frac{108\,x1}{17}$$

```
> plot3d(17*(y2^2-z2^2)/108,y2=-1..1,z2=-1..1);
```




```
>
```