O19-C10

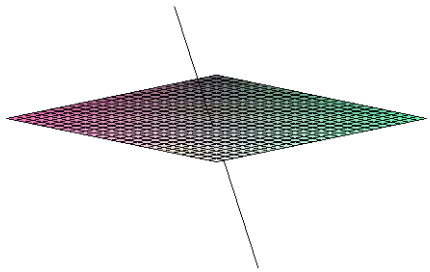
```
> restart;
> d1:=(x+y-2*z-1)/sqrt(6);
```

$$d1 := \frac{(x+y-2z-1)\sqrt{6}}{6}$$

```
> d2:=sqrt((y-2*z-1)^2+(z-x+1)^2+(2*x-y-1)^2)/sqrt(6);
```

$$d2 := \frac{\sqrt{2y^2-4yz+5z^2+6z+3-2zx+5x^2-6x-4xy}\,\sqrt{6}}{6}$$

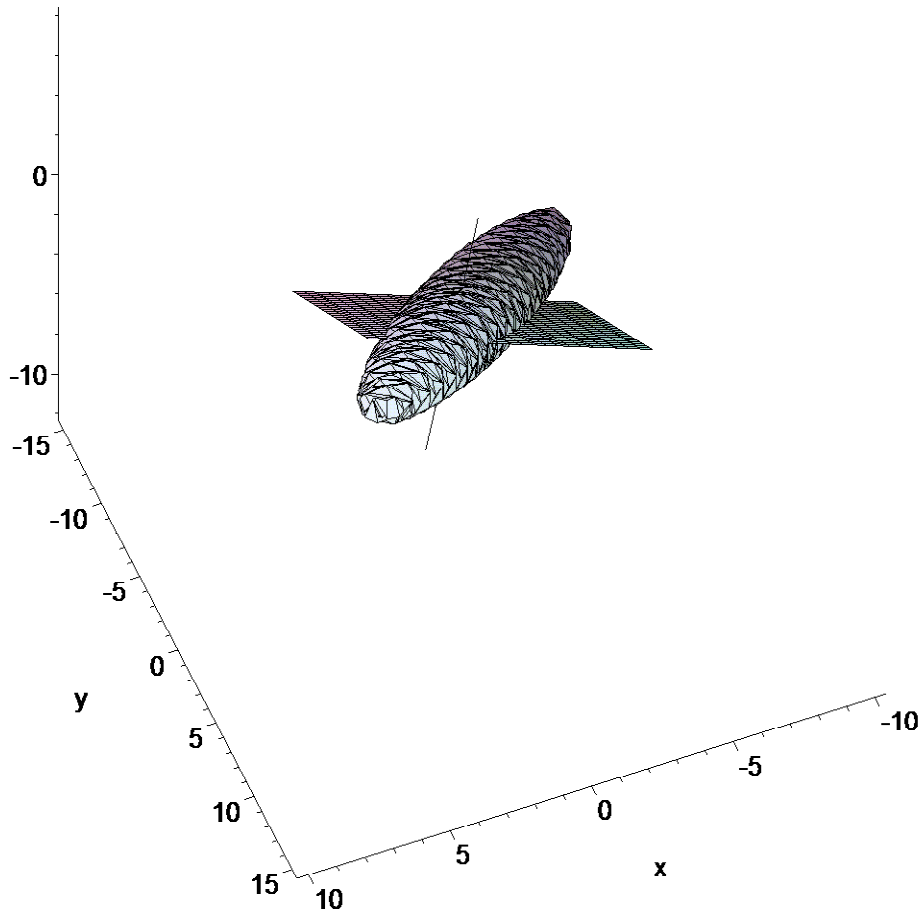
```
> with(plots):
Warning, the name changecoords has been redefined

> p:=plot3d((x+y-1)/2,x=-5..5,y=-5..5):
> d:=polygonplot3d([[-1,-3,2],[3,5,-2]]):
> display3d({p,d});
```



```
> S:=expand(d1^2+d2^2=5);
```

$$S := x^2 - \frac{1}{3}xy - zx - \frac{4}{3}x + \frac{1}{2}y^2 - \frac{4}{3}yz - \frac{1}{3}y + \frac{3}{2}z^2 + \frac{5}{3}z + \frac{2}{3} = 5$$

```
> surf:=implicitplot3d(S,x=-10..10,y=-15..15,z=-12..8,numpoints=15
  000):
> display3d({d,p,surf});
```




```
> gra:={diff(S,x),diff(S,y),diff(S,z)};
```

$$gra := \{2x - \frac{y}{3} - z - \frac{4}{3} = 0, -\frac{x}{3} + y - \frac{4z}{3} - \frac{1}{3} = 0, -x - \frac{4y}{3} + 3z + \frac{5}{3} = 0\}$$

```
> centre:=solve(gra);
```

$$centre := \{x = 0, z = -1, y = -1\}$$

```
> S1:=expand(subs({x=xx,y=-1+yy,z=-1+zz},S));
```

$$S1 := xx^2 - \frac{1}{3}xx\,yy - xx\,zz + \frac{1}{2}yy^2 - \frac{4}{3}yy\,zz + \frac{3}{2}zz^2 = 5$$

```
> with(LinearAlgebra):
> Q:=Matrix([[1,-1/6,-1/2],[-1/6,1/2,-2/3],[-1/2,-2/3,3/2]]);
```

$$Q := \begin{bmatrix} 1 & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{6} & \frac{1}{2} & \frac{-2}{3} \\ \frac{-1}{2} & \frac{-2}{3} & \frac{3}{2} \end{bmatrix}$$

```
> vp:=Eigenvalues(Q);evalf(%);
```

$$vp := \begin{bmatrix} 1 \\ 1+\frac{\sqrt{35}}{6} \\ 1-\frac{\sqrt{35}}{6} \end{bmatrix}$$

$$\begin{bmatrix} 1. \\ 1.986013297 \\ 0.0139867026 \end{bmatrix}$$

```
> volume:=4*Pi*5*sqrt(5)/3/sqrt(vp[1]*vp[2]*vp[3]);evalf(%);
```

$$volume := \frac{20\,\pi\sqrt{5}}{3\sqrt{\left(1+\frac{\sqrt{35}}{6}\right)\left(1-\frac{\sqrt{35}}{6}\right)}}$$

280.9925913

```
>
```