

[ O19-C07

[ > **restart:**

[ > **with(LinearAlgebra):**

[ > **a:=Matrix([[ -23,10,3,-11],[314,-126,-39,139],[-426,174,56,-187],[225,-92,-29,100]]);**

$$a := \begin{bmatrix} -23 & 10 & 3 & -11 \\ 314 & -126 & -39 & 139 \\ -426 & 174 & 56 & -187 \\ 225 & -92 & -29 & 100 \end{bmatrix}$$

[ > **u:=<1,2,3,4>;**

$$u := \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

[ > **a.u;**

$$\begin{bmatrix} -38 \\ 501 \\ -658 \\ 354 \end{bmatrix}$$

[ > **P:=CharacteristicPolynomial(a,X);**

$$P := X^4 - 7X^3 + 18X^2 - 20X + 8$$

[ > **Eigenvectors(a);**

$$\begin{bmatrix} 1 \\ 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} \frac{-10}{91} & 1 & 0 & 0 \\ \frac{128}{91} & \frac{-25}{2} & 0 & 0 \\ \frac{-173}{91} & 17 & 0 & 0 \\ 1 & -9 & 0 & 0 \end{bmatrix}$$

[ > **v1:=<-8,103,-139,73>;v2:=a.v1:v3:=a.v2:**

[ > **b:=Matrix([v1,v2,v3]);**

$$b := \begin{bmatrix} -8 & -6 & -2 \\ 103 & 78 & 28 \\ -139 & -105 & -37 \\ 73 & 55 & 19 \end{bmatrix}$$

[ > **NullSpace(b);**

$$\left\{ \begin{bmatrix} 1 \\ -\frac{3}{2} \\ \frac{1}{2} \end{bmatrix} \right\}$$

[ > **aF:=Matrix([[0,-2],[1,3]]);**

$$aF := \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix}$$

> PF:=CharacteristicPolynomial(aF,X);

$$PF := X^2 - 3X + 2$$

> Eigenvectors(aF);

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 & -1 \\ 1 & 1 \end{bmatrix}$$

> w1:=<2,-13,17,-9>;w2:=a.w1:w3:=a.w2:w4:=a.w3:

> c:=Matrix([w1,w2,w3,w4]);

$$c := \begin{bmatrix} 2 & -26 & -102 & -294 \\ -13 & 352 & 1332 & 3792 \\ 17 & -479 & -1811 & -5155 \\ -9 & 253 & 957 & 2725 \end{bmatrix}$$

> NullSpace(c);

$$\left\{ \begin{bmatrix} -4 \\ 8 \\ -5 \\ 1 \end{bmatrix} \right\}$$

> aG:=Matrix([[0,0,4],[1,0,-8],[0,1,5]]);

$$aG := \begin{bmatrix} 0 & 0 & 4 \\ 1 & 0 & -8 \\ 0 & 1 & 5 \end{bmatrix}$$

> PG:=CharacteristicPolynomial(aG,X);

> Eigenvectors(aG);

$$PG := X^3 - 5X^2 + 8X - 4$$

$$\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 & 0 & 4 \\ -3 & 0 & -4 \\ 1 & 0 & 1 \end{bmatrix}$$

> aa:=a-2\*IdentityMatrix(4);

$$aa := \begin{bmatrix} -25 & 10 & 3 & -11 \\ 314 & -128 & -39 & 139 \\ -426 & 174 & 54 & -187 \\ 225 & -92 & -29 & 98 \end{bmatrix}$$

> K:=[op(NullSpace(aa^2))];x:=K[2];

$$K := \left[ \begin{bmatrix} 0 \\ 3 \\ 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix} \right]$$

$$x := \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix}$$

> d:=Matrix([x,a.x,a.a.x]);NullSpace(d);

$$d := \begin{bmatrix} 1 & -16 & -68 \\ 1 & 227 & 904 \\ -1 & -308 & -1228 \\ 0 & 162 & 648 \end{bmatrix}$$

$$\left\{ \begin{bmatrix} 4 \\ -4 \\ 1 \end{bmatrix} \right\}$$

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> aH:=Matrix([[0,-4],[1,4]]);
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$$aH := \begin{bmatrix} 0 & -4 \\ 1 & 4 \end{bmatrix}$$

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> PH:=CharacteristicPolynomial(aH,X);
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> Eigenvectors(aH);
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$$PH := X^2 - 4X + 4$$

$$\begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix}$$

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