

[O19-C02

[> **restart;**

> **P:=proc(n)**

local l,i,ll;

l:=[2,X];

for i from 2 to n do

l:=[op(l),X*l[i]-l[i-1]]

od;

ll:=expand(l);

return ll;

end;

P := proc(n)

local l, i, ll;

l := [2, X];

for i from 2 to n do *l := [op(l), X*l[i] - l[i - 1]]* **end do;**

ll := expand(l);

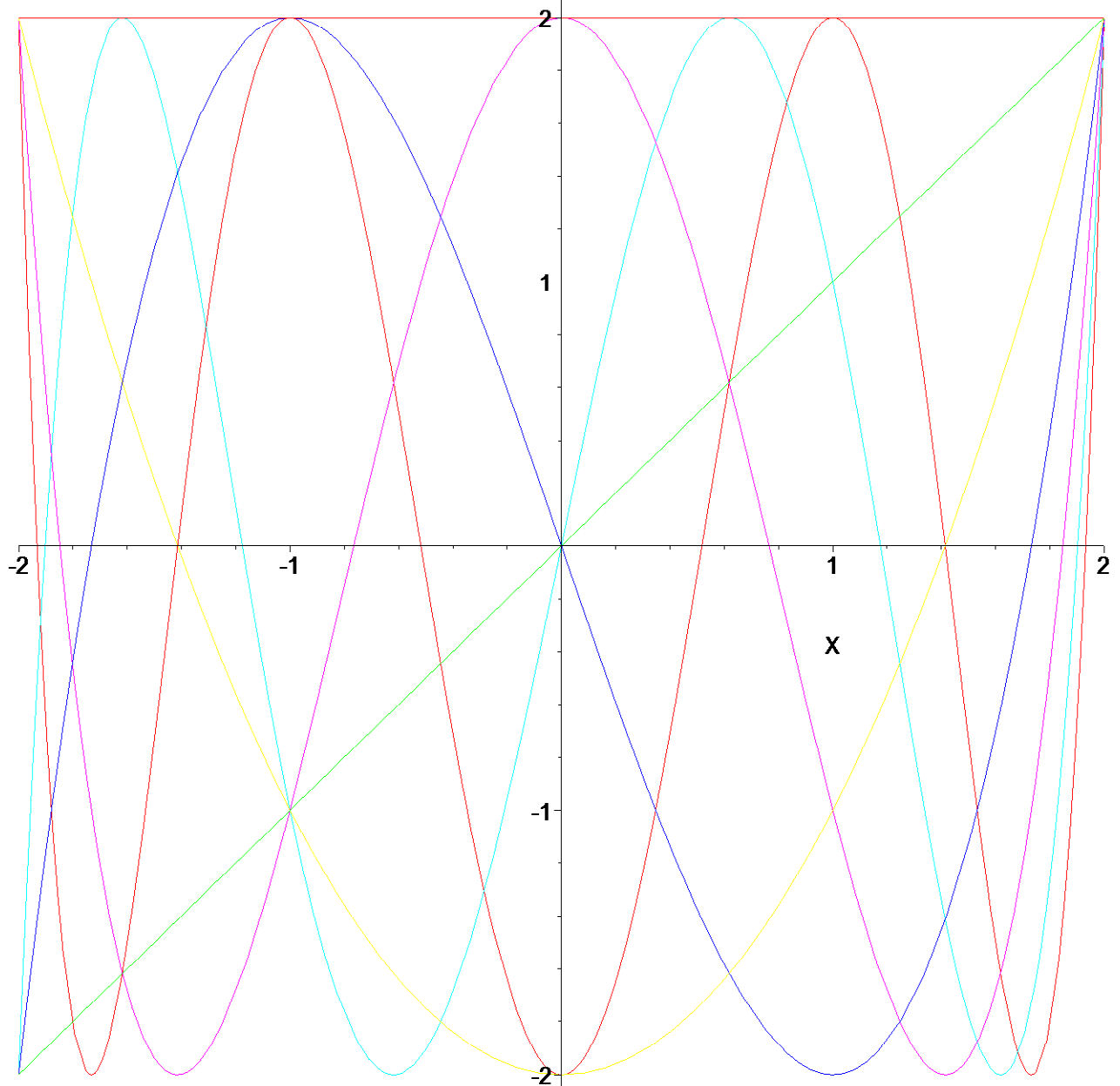
return ll

end proc

> **S:=P(6);**

$S := [2, X, X^2 - 2, X^3 - 3X, X^4 - 4X^2 + 2, X^5 - 5X^3 + 5X, X^6 - 6X^4 + 9X^2 - 2]$

> **plot(S,X=-2..2);**



```
> ps:=(a,b)->int(a*b/sqrt(4-X^2),X=-2..2);
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$$ps := (a, b) \rightarrow \int_{-2}^2 \frac{a b}{\sqrt{4 - X^2}} dX$$

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> with(LinearAlgebra):
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> m:=Matrix(7,7,(i,j)->ps(S[i],S[j]));
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$$m := \begin{bmatrix} 4\pi & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\pi & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\pi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\pi & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\pi & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\pi & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2\pi \end{bmatrix}$$

```
> GS:=proc(l,k)
```

```

local lo,v,i,N,coef;
N:=a->sqrt(ps(a,a));
lo:=[l[1]/(N(l[1]))];
for i from 2 to k do
    coef:=[seq(ps(lo[j],l[i]),j=1..i-1)];
    v:=l[i]-sum('coef[j]*lo[j]','j'=1..i-1);
    lo:=[op(lo),v/N(v)]
od;
return lo;
end;

```

GS := proc(l, k)

local lo, v, i, N, coef;

N := a → sqrt(ps(a, a));

lo := [l[1] / N(l[1])];

for i from 2 to k do

coef := [seq(ps(lo[j], l[i]), j = 1 .. i - 1)];

v := l[i] - sum('coef[j]*lo[j]', 'j' = 1 .. i - 1);

lo := [op(lo), v / N(v)]

end do;

return lo

end proc

> l:=[seq(X^i,i=0..6)]:k:=7;GS(l,k);

$$\begin{aligned}
 & k := 7 \\
 & \left[\frac{1}{\sqrt{\pi}}, \frac{X\sqrt{2}}{2\sqrt{\pi}}, \frac{(X^2-2)\sqrt{2}}{2\sqrt{\pi}}, \frac{(X^3-3X)\sqrt{2}}{2\sqrt{\pi}}, \frac{(X^4-4X^2+2)\sqrt{2}}{2\sqrt{\pi}}, \frac{(X^5-5X^3+5X)\sqrt{2}}{2\sqrt{\pi}}, \right. \\
 & \left. \frac{(X^6-6X^4+9X^2-2)\sqrt{2}}{2\sqrt{\pi}} \right]
 \end{aligned}$$

> for i from 1 to 7 do a:=[solve(S[i])];print(a,evalf(a)) od:

$$\begin{aligned}
 & [1, [] \\
 & [0], [0.] \\
 & [\sqrt{2}, -\sqrt{2}], [1.414213562, -1.414213562] \\
 & [0, \sqrt{3}, -\sqrt{3}], [0., 1.732050808, -1.732050808] \\
 & [-\sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2}}, -\sqrt{2-\sqrt{2}}, \sqrt{2-\sqrt{2}}], \\
 & [-1.847759065, 1.847759065, -0.7653668650, 0.7653668650] \\
 & \left[0, -\frac{\sqrt{10+2\sqrt{5}}}{2}, \frac{\sqrt{10+2\sqrt{5}}}{2}, -\frac{\sqrt{10-2\sqrt{5}}}{2}, \frac{\sqrt{10-2\sqrt{5}}}{2} \right], \\
 & [0., -1.902113032, 1.902113032, -1.175570504, 1.175570504] \\
 & \left[\sqrt{2}, -\sqrt{2}, -\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}, \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}, -\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}, \frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right],
 \end{aligned}$$

```

[ 1.414213562, -1.414213562, -1.931851653, 1.931851653, -0.5176380910, 0.5176380910]
> for i from 1 to 6 do
  a:=[seq(eval(2*cos((1+2*k)*Pi/2/i)),k=0..i-1)]:print(a,evalf(a))
od:

[0], [0.]
[ $\sqrt{2}$ ,  $-\sqrt{2}$ ], [1.414213562, -1.414213562]
[ $\sqrt{3}$ , 0,  $-\sqrt{3}$ ], [1.732050808, 0., -1.732050808]
[ $2 \cos\left(\frac{\pi}{8}\right)$ ,  $2 \cos\left(\frac{3\pi}{8}\right)$ ,  $-2 \cos\left(\frac{3\pi}{8}\right)$ ,  $-2 \cos\left(\frac{\pi}{8}\right)$ ],
[1.847759065, 0.7653668650, -0.7653668650, -1.847759065]
[ $2 \cos\left(\frac{\pi}{10}\right)$ ,  $2 \cos\left(\frac{3\pi}{10}\right)$ , 0,  $-2 \cos\left(\frac{3\pi}{10}\right)$ ,  $-2 \cos\left(\frac{\pi}{10}\right)$ ],
[1.902113033, 1.175570504, 0., -1.175570504, -1.902113033]
[ $2 \cos\left(\frac{\pi}{12}\right)$ ,  $\sqrt{2}$ ,  $2 \cos\left(\frac{5\pi}{12}\right)$ ,  $-2 \cos\left(\frac{5\pi}{12}\right)$ ,  $-\sqrt{2}$ ,  $-2 \cos\left(\frac{\pi}{12}\right)$ ],
[1.931851653, 1.414213562, 0.5176380902, -0.5176380902, -1.414213562, -1.931851653]
>
[ >

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