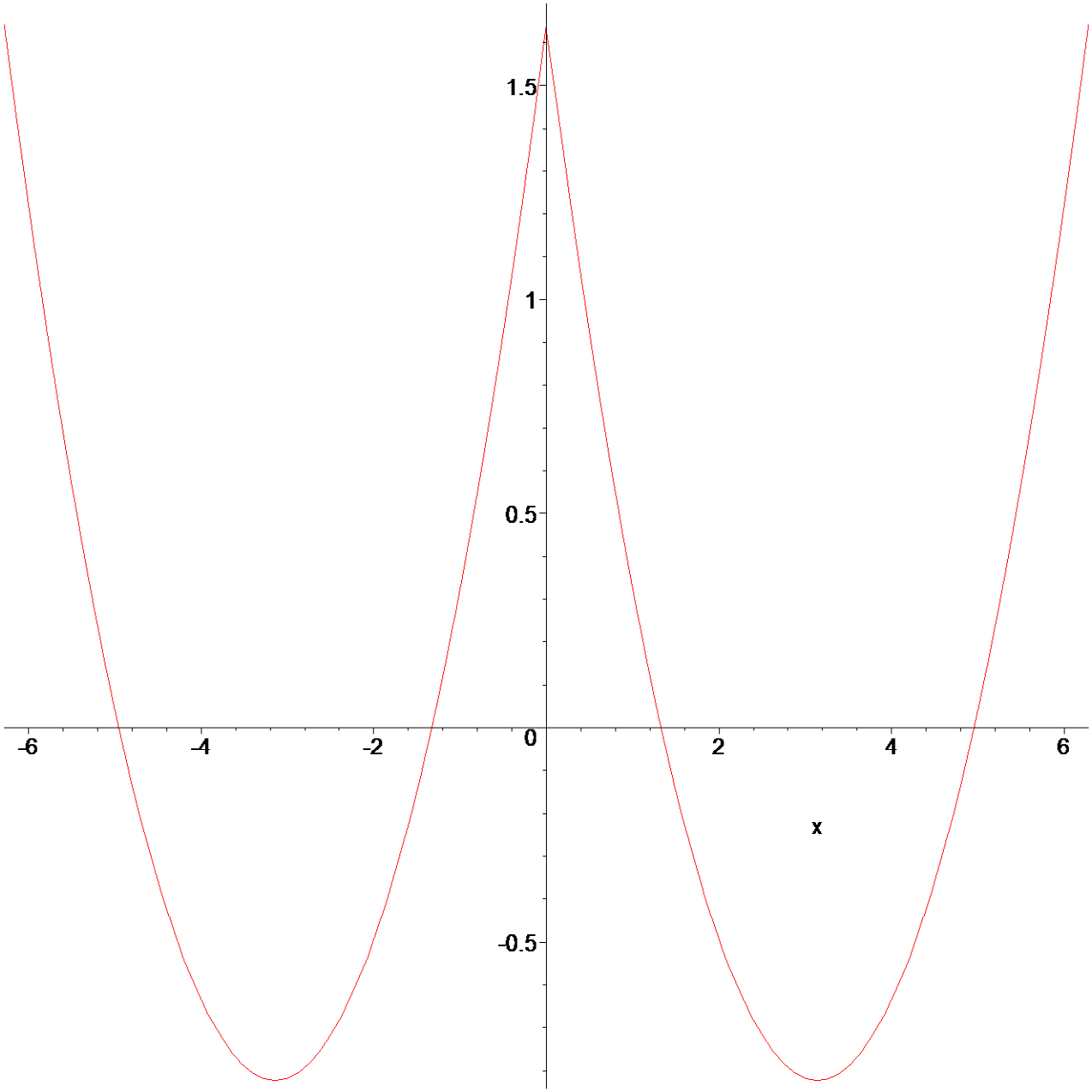
O19-096

```
> restart:
> s:=p->sum(cos(n*x)/n^2,n=1..p);f:=limit(s(p),p=infinity);
```

$$s := p \to \sum_{n=1}^{p} \frac{\cos(n\,x)}{n^2}$$

$$f := \lim_{p \to \infty} \sum_{n=1}^{p} \frac{\cos(n\,x)}{n^2}$$

```
> plot(s(1000),x=-2*Pi..2*Pi);
```




```
> edo:=p->diff(y(x),x$2)+y(x)/4=s(p):ic:=y(0)=1,D(y)(0)=1;
```

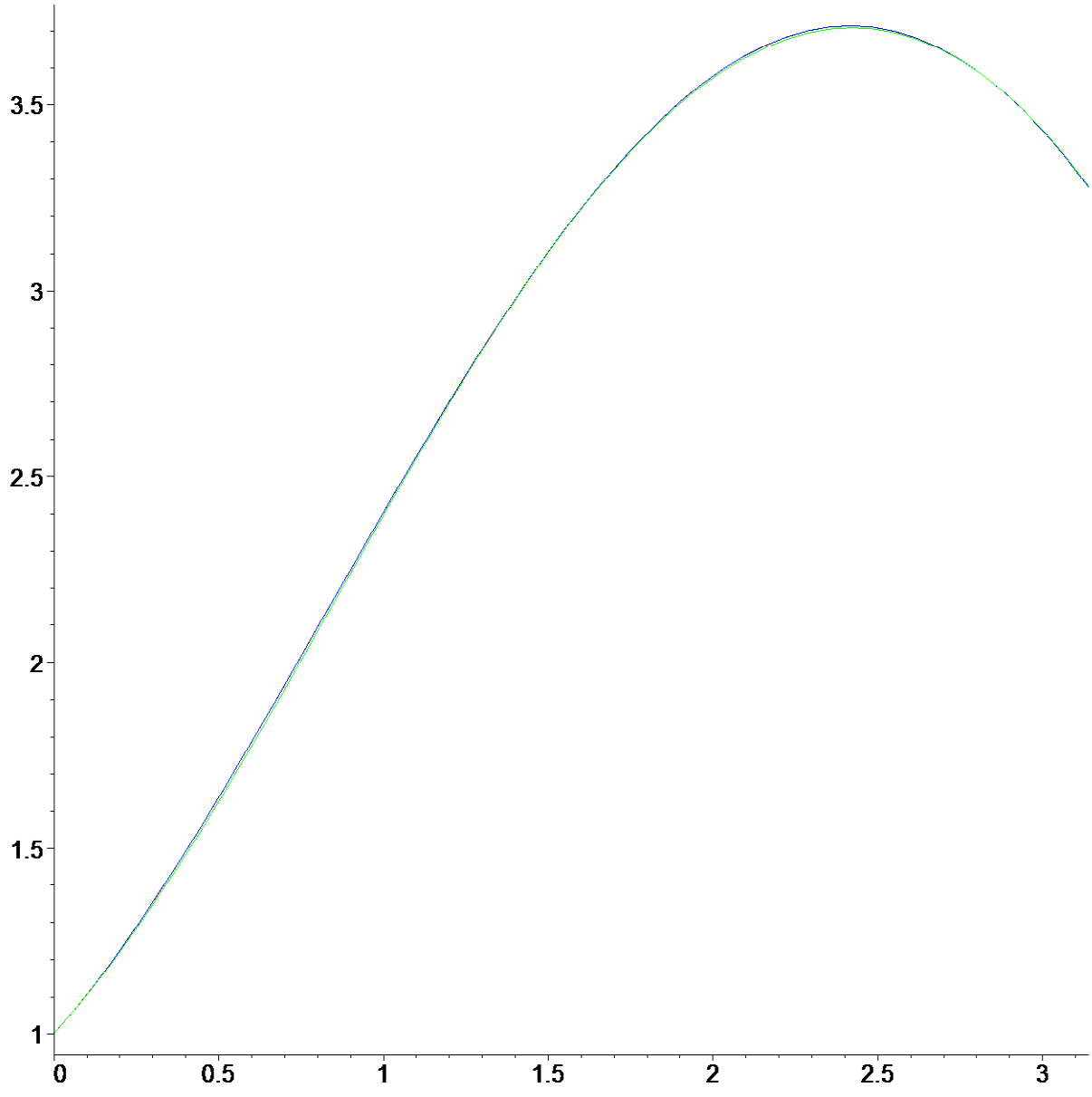
$$ic := y(0) = 1, D(y)(0) = 1$$

```
> sol0:=dsolve({edo(3),ic},y(x));y0:=subs(sol0,y(x)):
  sol1:=dsolve({edo(10),ic},y(x));y1:=subs(sol1,y(x)):
  sol2:=dsolve({edo(20),ic},y(x)):y2:=subs(sol2,y(x)):
```

$$sol0 := y(x) = 2\sin\left(\frac{x}{2}\right) + \frac{152}{63}\cos\left(\frac{x}{2}\right) - \frac{136}{105}\cos(x) - \frac{2}{15}\cos(x)^2 + \frac{1}{15} - \frac{16}{315}\cos(x)^3$$

$$sol1 := y(x) = 2\sin\left(\frac{x}{2}\right) + \frac{768391}{317520}\cos\left(\frac{x}{2}\right) + \frac{73714423}{1163962800} - \frac{1024}{26163}\cos(x)^9 - \frac{512}{9975}\cos(x)^{10}$$
$$+ \frac{567296}{9258795}\cos(x)^7 + \frac{1096}{11305}\cos(x)^8 - \frac{3275072}{72747675}\cos(x)^5 - \frac{30928}{415701}\cos(x)^6$$
$$+ \frac{18724}{2909907}\cos(x)^4 - \frac{1028672}{43648605}\cos(x)^3 - \frac{300636}{230945}\cos(x) - \frac{1640354}{14549535}\cos(x)^2$$

```
> plot([y0,y1,y2],x=0..Pi,color=[green,blue,red]);
```




```
> ff:=x->sum(cos(n*x)/n^2,n=1..infinity);
```

$$ff := x \to \sum_{n=1}^{\infty} \frac{\cos(n\,x)}{n^2}$$

```
> u:=ff(0);w:=-ff(0)/2;v:=-ff(0)/8;a:=0:b:=Pi/2:c:=Pi:
```

$$u := \frac{\pi^2}{6}$$

$$w := -\frac{\pi^2}{12}$$

$$v := -\frac{\pi^2}{48}$$

```
> P:=u*(x-b)*(x-c)/(a-b)/(a-c)+v*(x-a)*(x-c)/(b-a)/(b-c)+w*(x-a)*(
  x-b)/(c-a)/(c-b);
```

$$P := \frac{\left(x-\frac{\pi}{2}\right)(x-\pi)}{3} + \frac{x\,(x-\pi)}{12} - \frac{x\left(x-\frac{\pi}{2}\right)}{6}$$

```
> a:=n->int(P*cos(n*x),x=0..Pi)*2/Pi;
```

$$a := n \to \frac{2}{\pi}\int_0^{\pi} P\cos(n\,x)\,dx$$

```
> assume(n,integer):aa:=n^2*simplify(a(n));
```

$$aa := 1$$

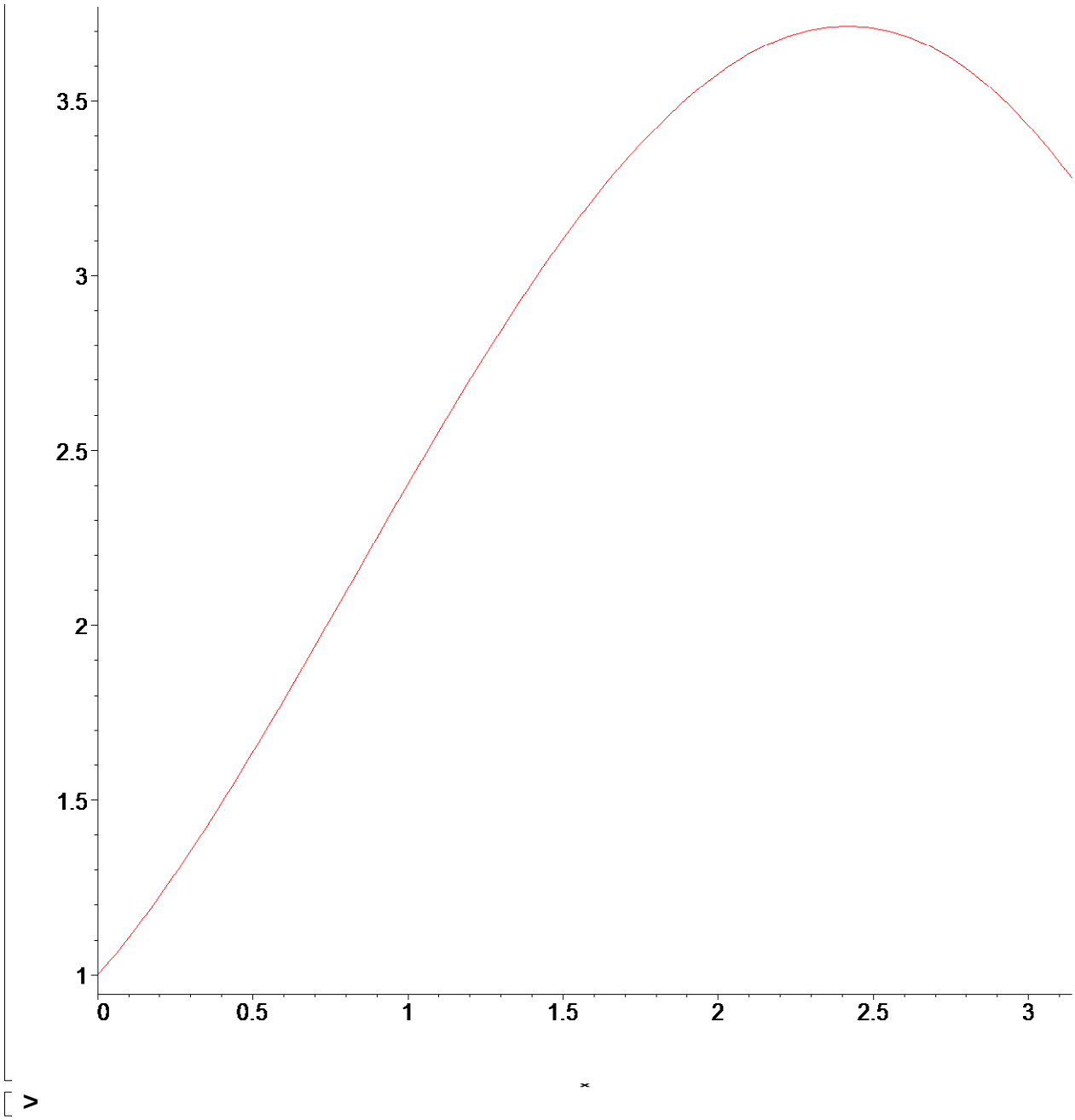
```
> edo1:=diff(y(x),x$2)+y(x)/4=P;
```

$$edo1 := \left(\frac{d^2}{dx^2}\,y(x)\right) + \frac{1}{4}\,y(x) = \frac{\left(x-\frac{\pi}{2}\right)(x-\pi)}{3} + \frac{x\,(x-\pi)}{12} - \frac{x\left(x-\frac{\pi}{2}\right)}{6}$$

```
> sol:=dsolve({edo1,ic},y(x));yy:=subs(sol,y(x)):
```

$$sol := y(x) = \sin\left(\frac{x}{2}\right)(4\,\pi+2) + \cos\left(\frac{x}{2}\right)\left(-\frac{2\,\pi^2}{3}+9\right) - 8 + \frac{2\,\pi^2}{3} - 2\,x\,\pi + x^2$$

```
> plot(yy,x=0..Pi);
```


3.5
3
2.5
2
1.5
1
0
0.5
1
1.5
2
2.5
3
x
V