

[O19-091

[> **restart;**

[> **a:=x->int(x*(1-t)/(1-x*t*(1-t))^2,t=0..1);**

$$a := x \rightarrow \int_0^1 \frac{x(1-t)}{(1-x t(1-t))^2} dt$$

[> **a(1);**

$$\frac{1}{3} + \frac{2\sqrt{3}\pi}{27}$$

[> **b:=x->int(x*(1-t)/(1-x*t*(1-t))^2,t);**

$$b := x \rightarrow \int \frac{x(1-t)}{(1-x t(1-t))^2} dt$$

[> **assume(x>-4);assume(x<4);**

[> **f:=b(x);**

$$f := x \rightarrow \left(\frac{x t - x + 2}{(4x - x^2)(1 - x t + x t^2)} - \frac{2x \operatorname{arctanh}\left(\frac{-x + 2x t}{\sqrt{-4x + x^2}}\right)}{(4x - x^2)\sqrt{-4x + x^2}} \right)$$

[> **x:='x';**

$x := x$

[> **c:=u->int(4*cos(u)*(1-t)/(1-4*cos(u)*t*(1-t))^2,t);**

$$c := u \rightarrow \int \frac{4 \cos(u)(1-t)}{(1-4 \cos(u)t(1-t))^2} dt$$

[> **g:=c(u);**

$$g := \frac{16 \cos(u)^2 t}{(16 \cos(u) - 16 \cos(u)^2)(1 - 4 \cos(u)t + 4 \cos(u)t^2)} - \frac{16 \cos(u)^2}{(16 \cos(u) - 16 \cos(u)^2)(1 - 4 \cos(u)t + 4 \cos(u)t^2)} + \frac{8 \cos(u)}{(16 \cos(u) - 16 \cos(u)^2)(1 - 4 \cos(u)t + 4 \cos(u)t^2)} + \frac{8 \cos(u)^2 \operatorname{arctan}\left(\frac{1 - 4 \cos(u) + 8 \cos(u)t}{4 \sqrt{\cos(u) - \cos(u)^2}}\right)}{(16 \cos(u) - 16 \cos(u)^2)\sqrt{\cos(u) - \cos(u)^2}}$$

[> **h:=simplify(subs(t=1,g)-subs(t=0,g));**

$$h := - \frac{\cos(u) \left(\operatorname{arctan}\left(\frac{\cos(u)}{\sqrt{-\cos(u)(-1 + \cos(u))}}\right) + \sqrt{-\cos(u)(-1 + \cos(u))} \right)}{(-1 + \cos(u))\sqrt{-\cos(u)(-1 + \cos(u))}}$$

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> k:=simplify(subs(cos(u)=x/4,h));
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$$k := -\frac{x \left(4 \arctan\left(\frac{x}{\sqrt{-x(-4+x)}}\right) + \sqrt{-x(-4+x)} \right)}{(-4+x)\sqrt{-x(-4+x)}}$$

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> s1:=subs(x=1,k);evalf(%);
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$$s1 := \frac{1}{9} \left(4 \arctan\left(\frac{\sqrt{3}}{3}\right) + \sqrt{3} \right) \sqrt{3}$$

0.7363998590

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> s2:=subs(x=1,diff(k,x));evalf(%);
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$$s2 := \frac{1}{9} \left(4 \arctan\left(\frac{\sqrt{3}}{3}\right) + \sqrt{3} \right) \sqrt{3} + \frac{1}{3}$$

1.069733192

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> u:=n->(n!)^2/(2*n)!;
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$$u := n \rightarrow \frac{(n!)^2}{(2n)!}$$

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> ss1:=evalf(sum(u(n),n=1..infinity));
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$$ss1 := 0.7363998585$$

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> ss2:=evalf(sum(n*u(n),n=1..infinity));
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$$ss2 := 1.069733192$$

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>
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