1. $\Gamma' \subset \Gamma$ en injectant $x(t), y(t)$ de Γ' dans l'équation de Γ.
   Inversement, Γ donne $(x+y)^2+3y^2 = 1$ donc, pour tout point de Γ, il existe $t \in \mathbb{R}$ tel que $x+y = \cos t$ et $\sqrt{3}y = \sin t$ donc ce point est sur Γ'.
2. On sait $R = \dfrac{\mathrm{d}\alpha}{\mathrm{d}s}$ où α est l'angle polaire du vecteur tangent (modulo π) et s un paramétrage normal (ie une abscisse curviligne).
   Pour Γ' : $R1 = \dfrac{\mathrm{d}\left(\arctan(y'(t)/x'(t))\right) / \mathrm{d}t}{\sqrt{x'^2(t) + y'^2(t)}}$.
   On sait aussi que $\dfrac{\mathrm{d}T}{\mathrm{d}s} = cN$ où (T, N) est le repère de Frenet et c est la courbure.
   Pour Γ : On trouve T en normant un vecteur normal au gradient de la fonction associée.
   On sait que $T = (\dfrac{\mathrm{d}x}{\mathrm{d}s}, \dfrac{\mathrm{d}y}{\mathrm{d}s})$.
   On cherche la première coordonnée de $\dfrac{\mathrm{d}T}{\mathrm{d}s}$, d'abord en fonction de $\dfrac{\mathrm{d}x}{\mathrm{d}s}$ et $\dfrac{\mathrm{d}y}{\mathrm{d}s}$ puis en fonction de x et y d'où c en divisant par la première coordonnée de N.
   En un point de paramètre t, les deux résultats coïncident ! ! !

O18-C047

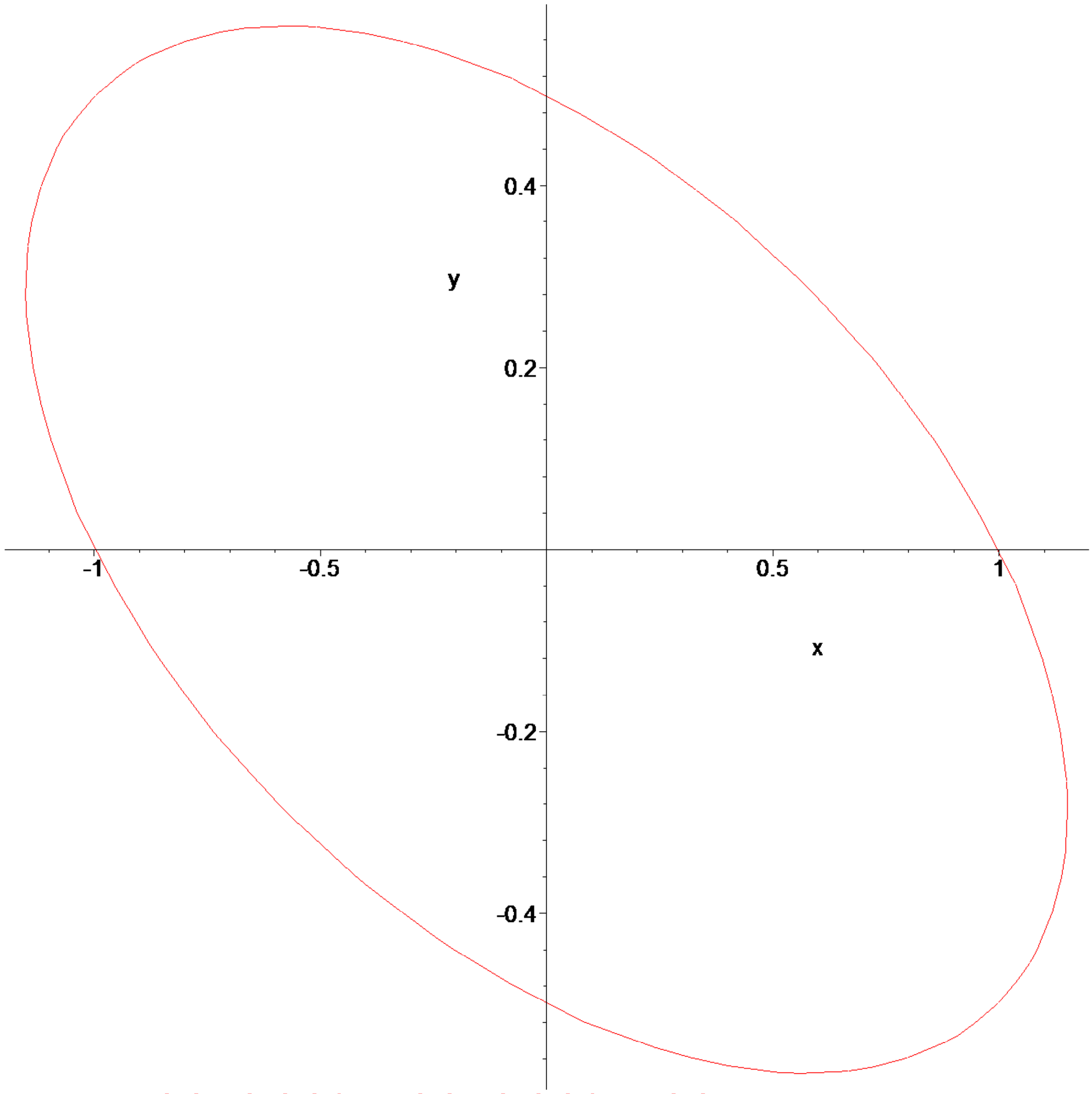
```
> restart;
> with(plots);
Warning, the name changecoords has been redefined
```

[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *cylinderplot*, *densityplot*, *display*, *display3d*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *graphplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *interactive*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *odeplot*, *pareto*, *plotcompare*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra_supported*, *polyhedraplot*, *replot*, *rootlocus*, *semilogplot*, *setoptions*, *setoptions3d*, *spacecurve*, *sparsematrixplot*, *sphereplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*]

```
> G:=x^2+2*x*y+4*y^2-1;
```

$$G := x^2 + 2\,x\,y + 4\,y^2 - 1$$

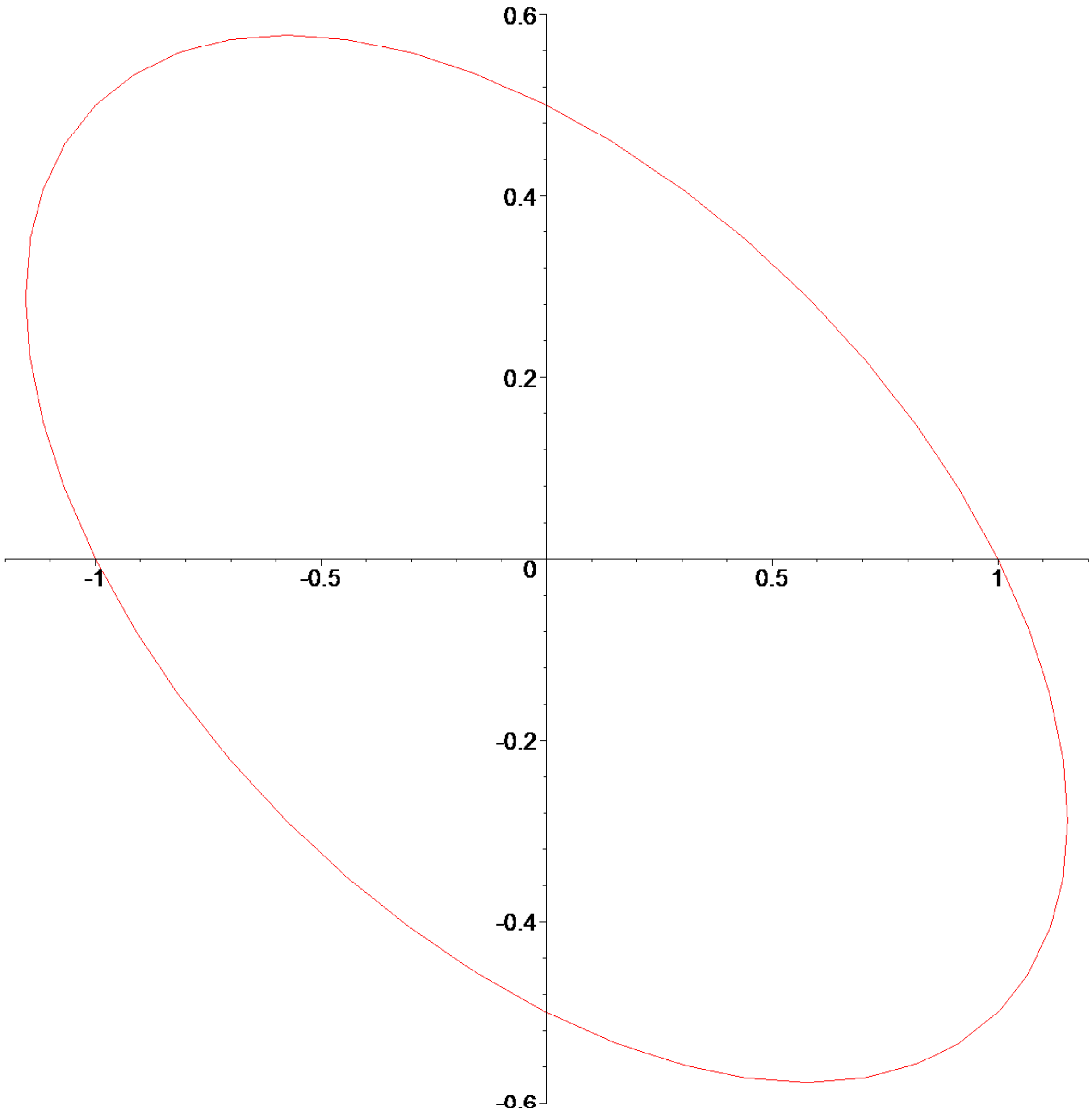
```
> implicitplot(G,x=-2..2,y=-1..1);
```




```
> G1:=<cos(t)-sin(t)/sqrt(3),sin(t)/sqrt(3)>;
```

$$G1 := \begin{bmatrix} \cos(t) - \frac{1}{3}\sin(t)\sqrt{3} \\ \frac{1}{3}\sin(t)\sqrt{3} \end{bmatrix}$$

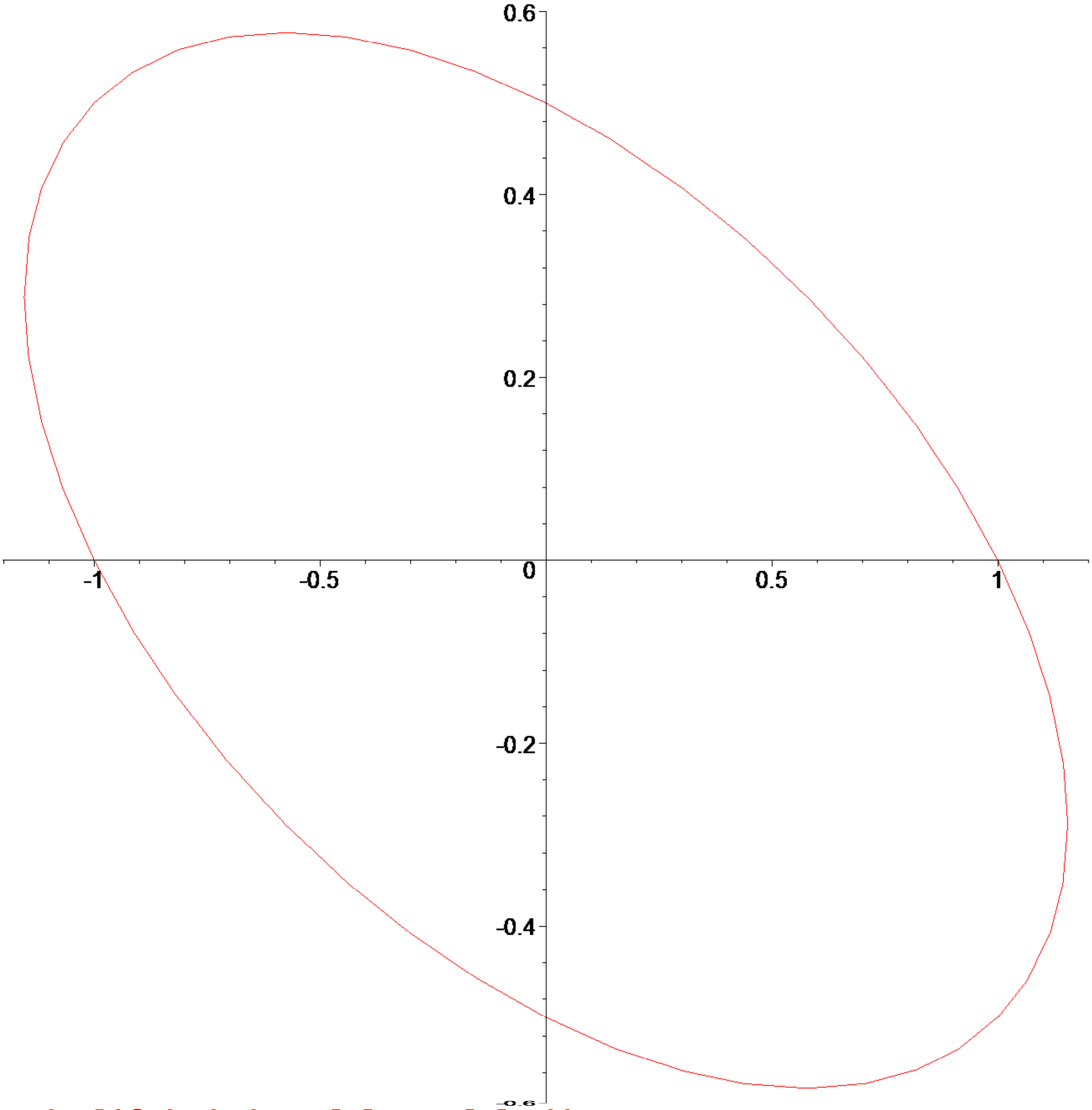
```
> plot([G1[1],G1[2],t=-Pi..Pi]);
```




```
> z:=G1[1]+I*G1[2];
```

$$z := \cos(t) - \frac{1}{3}\sin(t)\sqrt{3} + \frac{1}{3} I \sin(t)\sqrt{3}$$

```
> complexplot(z,t=-Pi..Pi);
```




```
> simplify(subs(x=G1[1],y=G1[2],G));
```

$$0$$

```
> G1prim:=<diff(G1[1],t),diff(G1[2],t)>;G1second:=<diff(G1prim[1],
  t),diff(G1prim[2],t)>;
```

$$G1prim := \begin{bmatrix} -\sin(t) - \frac{1}{3}\cos(t)\sqrt{3} \\ \frac{1}{3}\cos(t)\sqrt{3} \end{bmatrix}$$

$$G1second := \begin{bmatrix} -\cos(t) + \frac{1}{3}\sin(t)\sqrt{3} \\ -\frac{1}{3}\sin(t)\sqrt{3} \end{bmatrix}$$

```
> with(LinearAlgebra);
```

[*Add*, *Adjoint*, *BackwardSubstitute*, *BandMatrix*, *Basis*, *BezoutMatrix*, *BidiagonalForm*, *BilinearForm, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, ConditionNumber, ConstantMatrix, ConstantVector, CreatePermutation, CrossProduct, DeleteColumn, DeleteRow, Determinant, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, GaussianElimination, GenerateEquations, GenerateMatrix, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, LA_Main, LUDecomposition, LeastSquares, LinearSolve, Map, Map2, MatrixAdd, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, QRDecomposition, RandomMatrix, RandomVector, Rank, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, SubMatrix, SubVector, SumBasis, SylvesterMatrix, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip*]

```
> assume(t,real):c1:=simplify((-G1prim[1]*G1second[2]+G1prim[2]*G1second[1])/(Norm(G1prim,2))^(3));R1:=1/c1;
```

$$c1 := -\frac{3}{\left(3-\cos(t\sim)^2+2\sin(t\sim)\cos(t\sim)\sqrt{3}\right)^{(3/2)}}$$

$$R1 := -\frac{1}{3}\left(3-\cos(t\sim)^2+2\sin(t\sim)\cos(t\sim)\sqrt{3}\right)^{(3/2)}$$

```
> assume(x::real,y::real):tangent:=<x+4*y,-x-y>;T:=simplify(tangent/Norm(tangent,2));N:=<T[2],-T[1]>;
```

$$tangent := \begin{bmatrix} x\sim+4\,y\sim \\ -x\sim-y\sim \end{bmatrix}$$

$$T := \begin{bmatrix} \dfrac{x\sim+4\,y\sim}{\sqrt{2\,x\sim^2+10\,x\sim\,y\sim+17\,y\sim^2}} \\ -\dfrac{x\sim+y\sim}{\sqrt{2\,x\sim^2+10\,x\sim\,y\sim+17\,y\sim^2}} \end{bmatrix}$$

$$N := \begin{bmatrix} -\dfrac{x\sim+y\sim}{\sqrt{2\,x\sim^2+10\,x\sim\,y\sim+17\,y\sim^2}} \\ -\dfrac{x\sim+4\,y\sim}{\sqrt{2\,x\sim^2+10\,x\sim\,y\sim+17\,y\sim^2}} \end{bmatrix}$$

```
> dTxsurds:=simplify(diff(T[1],x)*T[1]+diff(T[1],y)*T[2]);
```

$$dTxsurds := -\frac{3\,(x\sim^3 + 3\,x\sim^2\,y\sim + 6\,x\sim\,y\sim^2 + 4\,y\sim^3)}{(2\,x\sim^2 + 10\,x\sim\,y\sim + 17\,y\sim^2)^2}$$

```
> c:=simplify(dTxsurds/N[1]);
```

$$c := \frac{3\,(x\sim^2 + 2\,x\sim\,y\sim + 4\,y\sim^2)}{(2\,x\sim^2 + 10\,x\sim\,y\sim + 17\,y\sim^2)^{(3/2)}}$$

```
> RR:=simplify(subs(x=G1[1],y=G1[2],1/c));
```

$$RR := \frac{1}{3}\,(3 - \cos(t\sim)^2 + 2\,\sin(t\sim)\,\cos(t\sim)\,\sqrt{3}\,)^{(3/2)}$$

```
> simplify(RR+R1);
```

$$0$$

```
>
```