1. $r_{2^p} = r_1$ pour tout p par récurrence.
2. $f_n(\mathbb{R}) = f_n([-\pi,\pi])$ et f_n est continue sur le compact $[-\pi,\pi]$ donc M_n existe.

   Parseval : $(\|f_n\|_2)^2 = \displaystyle\sum_0^{n-1} |r_k|^2 = n$ (=nombre de termes de la somme)

O18-904

```
> restart;
> r:=proc(n)
  local p;
  if n=0 then
         return 1
  elif irem(n,2)=0 then
         return r(n/2)
  else
       p:=iquo(n,2);
       return (-1)^p*r(p)
  fi;
  end;
```

r := **proc**(n)

local p;

if $n = 0$ **then return** 1 **elif** irem(n, 2) = 0 **then return** r(1 / 2*n) **else** p := iquo(n, 2); **return** (-1)^p*r(p) **end if**

end proc

```
> seq(r(n),n=0..30);rr:=array(0..30,[seq(r(n),n=0..30)]):
```

1, 1, 1, -1, 1, 1, -1, 1, 1, 1, 1, -1, -1, -1, 1, -1, 1, 1, 1, -1, 1, 1, -1, 1, -1, -1, -1, 1, 1, 1, -1

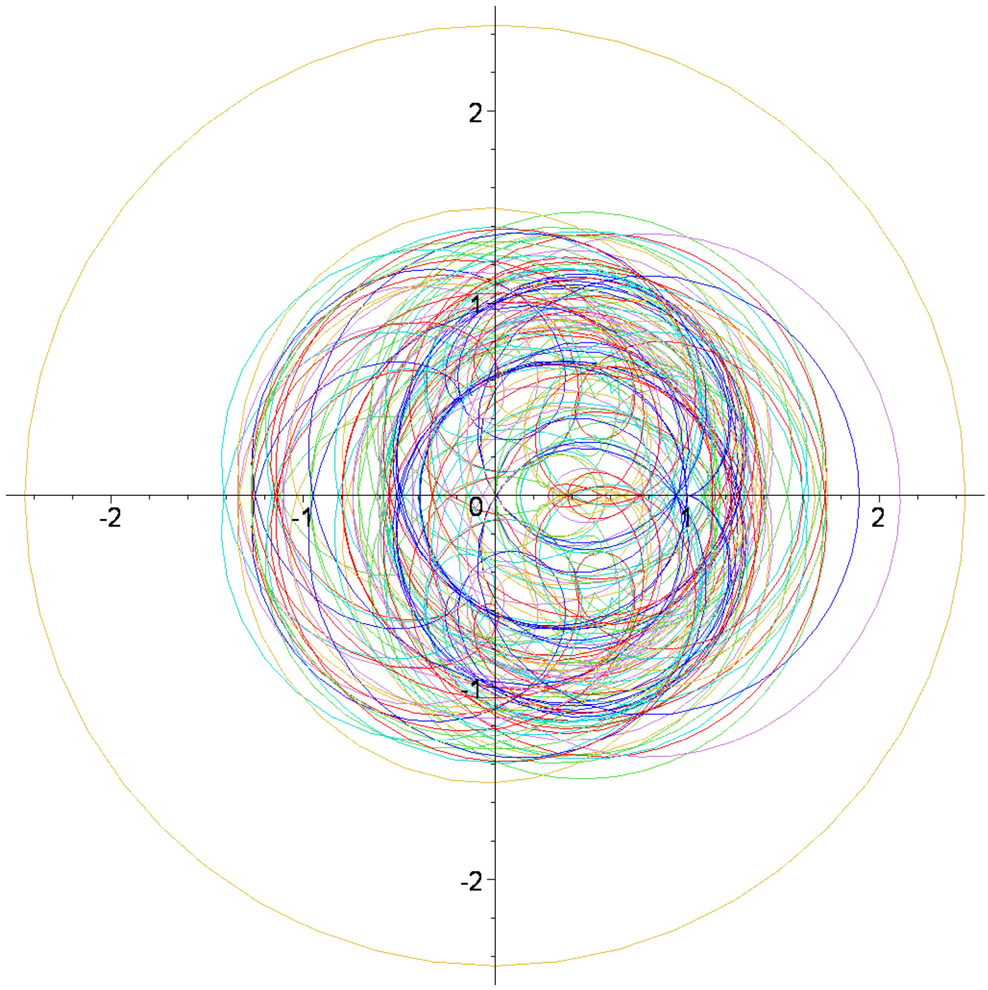
```
> S:=(t,n)->sum('rr[k]*exp(I*k*t)',k=0..n-1)/sqrt(n);
```

$$S := (t, n) \rightarrow \frac{\sum_{k=0}^{n-1} 'rr_k\, \mathbf{e}^{(k\, t\, I)}'}{\sqrt{n}}$$

```
> S(t,5);
```

$$\frac{1}{5}\left(1 + \mathbf{e}^{(t\, I)} + \mathbf{e}^{(2\, I\, t)} - \mathbf{e}^{(3\, I\, t)} + \mathbf{e}^{(4\, I\, t)}\right)\sqrt{5}$$

```
> with(plots):
> complexplot([seq(S(t,n),n=1..20),sqrt(6)*exp(I*t)],t=-Pi..Pi);
```



2
1
-2
-1
0
1
2
-1
-2

>