1. On applique la méthode de Schmidt : $u_1 = v_1/\|v_1\|$ puis $pro = \sum_1^{k-1}(v_k.u_i)u_i$ pour trouver le projeté orthogonal de v_k sur $Vect(u_1, ..., u_{k-1})$ et $u_k = (v_k - pro)/\|v_k - pro\|$.
2. $P = \sum_1^3(\cos .u_i)u_i$ et $\int_{\mathbb{R}} e^{-\pi x^2}(\cos x - P(x))^2 \, \mathrm{d}x = \min_{Q \in \mathbb{R}_2[x]} \int_{\mathbb{R}} e^{-\pi x^2}(\cos x - Q(x))^2 \, \mathrm{d}x$ se doit d'être faible. $e^{-\pi x^2}$ étant proche de 1 sur $[-1, 1]$ et de 0 ailleurs, $\int_{\mathbb{R}} e^{-\pi x^2}(\cos x - P(x))^2 \, \mathrm{d}x$ ressemble à $\int_{-1}^{1}(\cos x - P(x))^2 \, \mathrm{d}x$.

O16-C038

```
> restart:
> ps:=(f,g)->int(f*g*exp(-Pi*x^2),x=-infinity..infinity);
```

$$ps := (f, g) \to \int_{-\infty}^{\infty} f\, g\, e^{(-\pi x^2)}\, dx$$

```
> noorm:=f->sqrt(ps(f,f));noorm(x);
```

$$noorm := f \to \sqrt{\mathrm{ps}(f, f)}$$

$$\frac{\sqrt{2}}{2\sqrt{\pi}}$$

```
> schOrtho:=proc(V,n)
  local U,k,pro,coef;
  U[1]:=V[1]/noorm(V[1]);
  for k from 2 to n do
          coef:=[seq(ps(V[k],U[i]),i=1..k-1)];
          pro:=sum('coef[i]*U[i]','i'=1..k-1);
          U[k]:=(V[k]-pro)/noorm(V[k]-pro)
  od;
  return U;
  end;
```

schOrtho := **proc**(*V*, *n*)
local *U*, *k*, *pro*, *coef*;
U[1] := *V*[1] / noorm(*V*[1]);
for *k* **from** 2 **to** *n* **do**
coef := [seq(ps(*V*[*k*], *U*[*i*]), *i* = 1 .. *k* − 1)];
pro := sum('*coef*[*i*]**U*[*i*]', '*i*' = 1 .. *k* − 1);
U[*k*] := (*V*[*k*] − *pro*) / noorm(*V*[*k*] − *pro*)
end do;
return *U*
end proc

```
> V:=[1,x,x^2];U:=schOrtho(V,3);print(U);
```

$$V := [1, x, x^2]$$

$$U := U$$

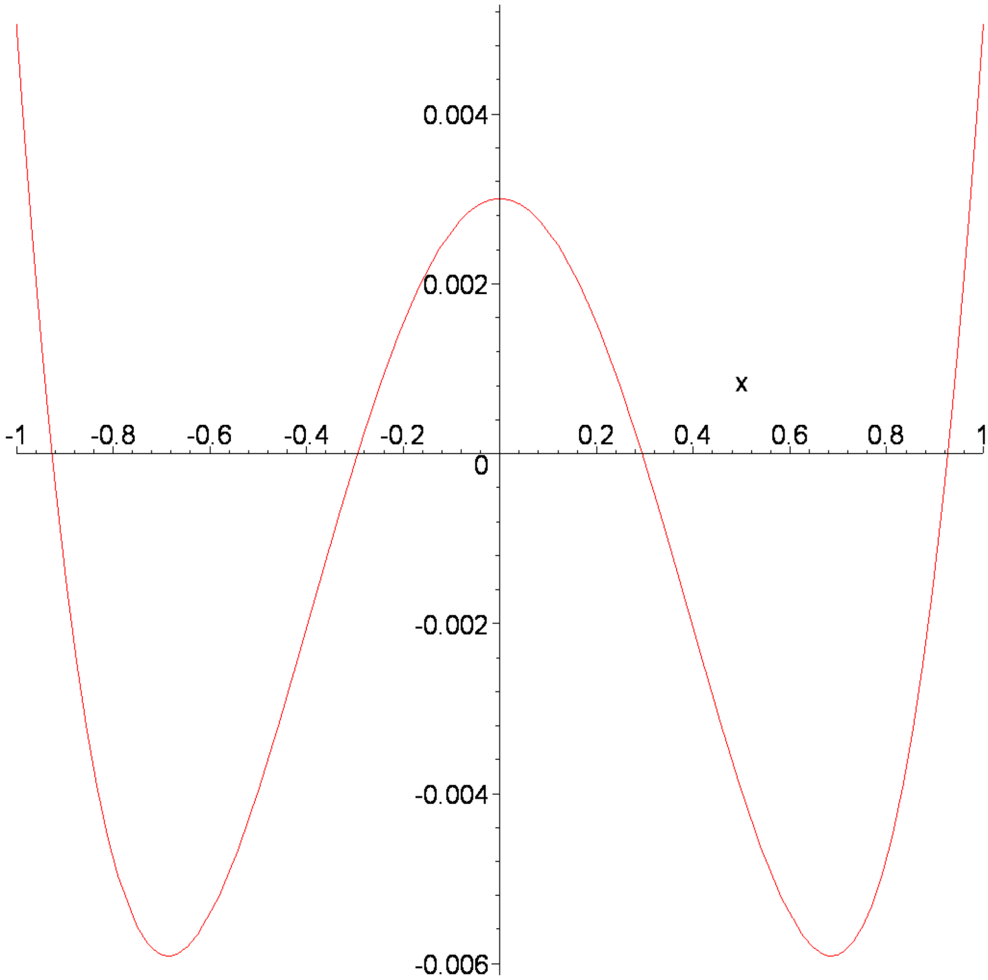
$$\mathrm{table}([1 = 1, 2 = x\sqrt{2}\sqrt{\pi}, 3 = \left(x^2 - \frac{1}{2\pi}\right)\sqrt{2}\,\pi])$$

```
> coef:=[seq(ps(cos(x),U[i]),i=1..3)];P:=sum('coef[i]*U[i]','i'=1..3);
```

$$coef := \left[\frac{1}{\left(e^{\left(\frac{1}{\pi}\right)}\right)^{(1/4)}}, 0, -\frac{1}{4}\frac{\sqrt{2}\, e^{\left(-\frac{1}{4\pi}\right)}}{\pi}\right]$$

$$P := \frac{1}{\left(e^{\left(\frac{1}{\pi}\right)}\right)^{(1/4)}} - \frac{1}{2} e^{\left(-\frac{1}{4\pi}\right)}\left(x^2 - \frac{1}{2\pi}\right)$$

```
> plot(cos(x)-P,x=-1..1);evalf(int(cos(x)-P,x=-1..1));
```



-0.003216103032

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